

About change of units

Question 8 from HW1

Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. If we measure temperature in degree Celsius and time in minute, the constant of proportionality  $k$  equals 0.3. The ambient temperature is  $T_A(t)$ . Write the differential equation that describes the time evolution of the temperature  $T$  of the object.

Approach 1:  $T, t$  are with units

$$k = 0.3 \text{ min}^{-1}$$

$$T_A(t) = 22 \cos\left(\frac{\pi t}{30 \text{ min}}\right) \text{ celsius}$$

$$\frac{dT}{dt} = \frac{0.3}{\text{min}} \times \left(22 \cos\left(\frac{\pi t}{30 \text{ min}}\right) \text{ celsius} - T\right)$$

$$\begin{aligned} \frac{dT}{dt} &= \frac{0.3}{\text{hour}} \times \left(22 \cos\left(\frac{\pi t}{30 \text{ hour}} \cdot \frac{\text{hour}}{\text{min}}\right) \text{ celsius} - T\right) \times \frac{\text{hour}}{\text{min}} \\ &= \frac{0.3}{\text{hour}} \times 60 \left(22 \cos\left(\frac{\pi t}{30 \text{ hour}} \cdot 60\right) \text{ celsius} - T\right) \end{aligned}$$

$$\frac{dT}{dt} = \frac{18}{\text{hour}} \left( 22 \cos \frac{2\pi t}{\text{hour}} \text{ celsius} - T \right)$$

(use  $A \text{ celsius} = \left(\frac{9}{5} A + 32\right) \text{ Fahrenheit}$ )

$$\begin{aligned} \frac{dT}{dt} &= \frac{18}{\text{hour}} \left( \frac{9}{5} \times 22 \cos \frac{2\pi t}{\text{hour}} \text{ Fahrenheit} + 32 \text{ Fahrenheit} - T \right) \\ &= \frac{18}{\text{hour}} \left( \frac{198}{5} \cos \frac{2\pi t}{\text{hour}} \text{ Fahrenheit} + 32 \text{ Fahrenheit} - T \right) \end{aligned}$$

Can just ignore all the units when writing down DE

Approach 2.  $T, t$  are without units

The temperature of the object is  $T$  celsius  
 $\tilde{T}$  Fahrenheit

$$\tilde{T} = \frac{9}{5} T + 32$$

$$\frac{dT}{dt} = 0.3 \left( 22 \cos \left( \frac{\pi}{30} t \right) - T \right)$$

$$t \text{ min} = \tilde{t} \text{ hour}$$

$$t = 60 \tilde{t}$$

$$\begin{aligned} \frac{dT}{d\tilde{t}} &= \frac{dT}{dt} \cdot \frac{dt}{d\tilde{t}} = 0.3 \left( 22 \cos \frac{\pi}{30} \tilde{t} \frac{t}{\tilde{t}} - T \right) \frac{dt}{d\tilde{t}} \\ &= 18 (22 \cos 2\pi \tilde{t} - T) \end{aligned}$$

$$\begin{aligned} \frac{d\tilde{T}}{d\tilde{t}} &= \frac{9}{5} \frac{dT}{d\tilde{t}} = \frac{9}{5} \times 18 (22 \cos 2\pi \tilde{t} - \frac{5}{9} (\tilde{T} - 32)) \\ &= 18 \left( \frac{198}{5} \cos 2\pi \tilde{t} + 32 - \tilde{T} \right) \end{aligned}$$