Calculate the integration of

$$I = \int e^{ax} \sin bx \mathrm{d}x$$

We proceed

$$\begin{aligned}
I &= \frac{1}{a} \int \sin bx \mathrm{d}e^{ax} \\
&= \frac{1}{a} \int \sin bx \mathrm{d}e^{ax} \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{1}{a} \int e^{ax} \mathrm{d} \sin bx \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a} \int e^{ax} \cos bx \mathrm{d}x \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} \int \cos bx \mathrm{d}e^{ax} \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx + \frac{b}{a^2} \int e^{ax} \mathrm{d} \cos bx \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx - \frac{b^2}{a^2} \int e^{ax} \sin bx \mathrm{d}x \\
&= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx - \frac{b^2}{a^2} I + C.
\end{aligned}$$

We then have

$$I + \frac{b^2}{a^2} I = \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx + C,$$

and then

$$\begin{aligned}
I &= \frac{a^2}{a^2 + b^2} \left( \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx \right) + C \\
&= \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C.
\end{aligned}$$

Calculate the integration of

$$J = \int e^{ax} \cos bx \mathrm{d}x$$

We proceed

$$
\begin{aligned}
J &= \frac{1}{a}\int \cos bx \mathrm{d}e^{ax} \\
&= \frac{1}{a}\int \cos bx \mathrm{d}e^{ax} \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{1}{a}\int e^{ax}\mathrm{d}\sin bx \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{b}{a}\int e^{ax}\sin bx \mathrm{d}x \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}\int \sin bx \mathrm{d}e^{ax} \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx - \frac{b}{a^2}\int e^{ax}\mathrm{d}\sin bx \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx - \frac{b^2}{a^2}\int e^{ax}\cos bx \mathrm{d}x \\
&= \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}J + C.
\end{aligned}
$$

We then have

$$
J + \frac{b^2}{a^2}J = \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx + C,
$$

and then

$$
\begin{aligned}
J &= \frac{a^2}{a^2+b^2}\left(\frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx\right) + C \\
&= \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C.
\end{aligned}
$$