1. (10 points) Solve the initial value problem

$$y'' + 6y' + 13y = 0, \quad y(0) = 2, \quad y'(0) = -2.$$

Write the solution in the form of $Re^{-\lambda t}\cos(\omega t - \varphi)$.

2. (8 points) Consider the linear homogeneous equation

$$t^2 y'' - 5ty' + 8y = 0$$

(a) (5 points) Find all values of $p$ such that $y(t) = t^p$ is a solution to the above equation.

(b) (3 points) Find the general solutions to the differential equation.

3. (10 points) Find the general solutions to

$$y'' - y = e^t + \cos t.$$

4. (10 points) An undamped mass spring system is released from equilibrium with a velocity of 6 m/s. The mass is 3 kg and it oscillates with an amplitude of 2 meters. There is no forcing. Find the spring constant $k$.

5. (12 points)  A 1kg mass is attached to a spring with spring constant 9 Newtons/m and is forced by an external force of $16 \sin 5t$ Newtons.  At time $t = 0$, the system is at equilibrium position $y = 0$ with initial velocity $y' = -2$m/s.  Formulate an initial value problem and solve it.  Write the solution as a product of two trigonometric functions.