

1. Find the Laplace Transforms of the following functions:

(a) $\frac{1}{2}t^3 + e^t \cos 5t$

(b) $f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ e^{-2t} - 1, & t \geq 1 \end{cases}$

(c) $f(t) = \begin{cases} \cos t, & 0 \leq t < \frac{\pi}{2} \\ 0, & t \geq \frac{\pi}{2} \end{cases}$

2. Find the Inverse Laplace Transforms of the following functions:

(a) $\frac{1}{s^2-5s+6}$

(b) $\frac{e^{-s}}{s^2+6s+10}$

(c) $\frac{1}{(s^2+1)(s^2+4)}$

(d) $\frac{3}{(s+1)^2(s+4)}$

(e) $\frac{e^{-2s}}{(s-1)^3}$

3. Solve the Initial Value Problem using Laplace Transform

$$y'' + 4y' + 13y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

4. Solve the Initial Value Problem

$$y' - 2y = \begin{cases} 0, & 0 \leq t < 2 \\ 4(t - 2), & t > 2 \end{cases}, \quad y(0) = 3.$$

5. Solve the Initial Value Problem

$$y'' + y = \delta(t - \pi) + \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 0.$$

Here y is a function of t . Sketch a graph of the solution.

6. Write the solution of the following IVP as a convolution integral

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = 0.$$