Math 307 - Section L
Win 2019
Exam 2
02/27/19

Name: Solutions

This exam contains 5 pages (including this cover page) and 4 problems. Put your first and last name on the top of this page.

You may *not* use your books, notes, or a **graphing** calculator on this exam.

You may use a **non-graphing** calculator on this exam.

You may use a single sheet of handwritten notes, with size not exceeding $11'' \times 8.5''$

Turn off all cellphones and electronic devices.

Do not open the exam until time.

Problem	Points	Score
1	16	
2	12	
3	14	
4	8	
Total:	50	

1. (16 points) The following parts are NOT related.

   (a) (4 points) It is known that $y(t) = \cos 3t$ is a solution to the initial value problem $y'' + 5y' - y = g(t)$, $y(0) = y_0$, $y'(0) = v_0$. Determine $g(t)$, y_0, v_0.

   $$g(t) = -9\cos 3t - 15\sin 3t - \cos 3t$$

   $$= -10\cos 3t - 15\sin 3t$$

   $y_0 = 1$, $v_0 = 0$

   (b) (4 points) Find the values of α and β such that $y(t) = \alpha t^\beta$ is a solution to $t^2y'' - ty' + y = t^2$.

   $$\alpha\beta(\beta-1)t^\beta - \alpha\beta t^\beta + \alpha t^\beta = t^2$$

   $$\alpha(\beta-1)^2 t^\beta = t^2$$

   $\beta = 2$, $\alpha(\beta-1)^2 = 1$

   $\alpha = 1$

   (c) [redacted]

   (d) (4 points) Write the function $u = 2\sqrt{3}\cos\frac{t}{2} + 2\sin\frac{t}{2}$ in the form of $R\cos(\omega t - \varphi)$.

   $$4\cos\left(\frac{1}{2}t - \frac{\pi}{6}\right)$$

2. (12 points) Solve the initial value problem

$$y'' + 4y' + 4y = 4t^2 + 8t + 6, \quad y(0) = 3, \quad y'(0) = -3.$$

homogeneous solns

$$C_1 e^{-2t} + C_2 t e^{-2t}$$

Special soln $Y = At^2 + Bt + C$

$$Y' = 2At + B$$

$$Y'' = 2A$$

$$2A + 4(2At + B) + 4(At^2 + Bt + C)$$

$$= 4At^2 + (8A + 4B)t + 2A + 4B + 4C$$

$$= 4t^2 + 8t + 6$$

$$A = 1, \quad B = 0, \quad C = 1$$

$$Y = t^2 + 1$$

$$y = C_1 e^{-2t} + C_2 t e^{-2t} + t^2 + 1$$

$$\left.\begin{array}{l} y(0) = C_1 + 1 = 3 \\ y'(0) = -2C_1 + C_2 = -3 \end{array}\right\} \begin{array}{l} C_1 = 2 \\ C_2 = 1 \end{array}$$

$$y = 2e^{-2t} + te^{-2t} + t^2 + 1$$

3. (14 points) Write down the general solutions for

$$y'' - 6y' = 3\cos 6t + 36t$$

homogeneous soln $y = C_1 e^{6t} + C_2$

① special solution for $3\cos 6t$

$Y = A\cos 6t + B\sin 6t$

$Y'' - 6Y' = -36A\cos 6t - 36B\sin 6t$
$+ 36A\sin 6t - 36B\cos 6t = 3\cos 6t$

$A - B = 0. \quad -36(A+B) = 3$

$A = B = -\frac{1}{24} \qquad Y = -\frac{1}{24}\cos 6t - \frac{1}{24}\sin 6t$

② special solution for $36t$

$Y = (At + B)\cdot t$

$Y'' - 6Y' = 2A - 6(2At + B) = 36t$

$-12A = 36 \qquad 2A - 6B = 0$

$A = -3. \quad B = -1 \qquad Y = -3t^2 - t$

$y = C_1 e^{6t} + C_2 - \frac{1}{24}\cos 6t - \frac{1}{24}\sin 6t$
$-3t^2 - t$

4. (8 points) Peter has a damped harmonic oscillator (without mass), but he did not know the spring constant or the damping coefficient. He prepared two objects A and B. The mass of Object A is 1 kilogram, and the mass of Object B is 4 kilograms. When peter attached Object A to the harmonic oscillater, the (quasi-) frequency of the oscillation he measured is 1 rad/s. When he used Object B, he measured the same frequency 1 rad/s. Determine the spring constant (in kg/s^2) and the damping coefficient (in kg/s), and write down the differential equation for the damped harmonic oscillator with Object A. You do NOT need to solve it.

$$my'' + \gamma y' + ky = 0$$

frequency $= \dfrac{\sqrt{4km - \gamma^2}}{2m} = 1$ when $m = 1$, $m = 4$

$$\frac{\sqrt{4k - \gamma^2}}{2} = 1$$

$$\frac{\sqrt{16k - \gamma^2}}{8} = 1$$

$$\begin{cases} 4k - \gamma^2 = 4 \\ 16k - \gamma^2 = 64 \end{cases}$$

$k = 5$, $\gamma = 4$

$$y'' + 4y' + 5y = 0$$