Basic ones are

(1) $$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,$$

(2) $$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,$$

(3) $$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta,$$

(4) $$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta,$$

The following identities can be derived from above ones

(5) $$\sin\theta + \sin\varphi \quad = 2\sin\tfrac{\theta+\varphi}{2}\cos\tfrac{\theta-\varphi}{2},$$

(6) $$\sin\theta - \sin\varphi \quad = 2\sin\tfrac{\theta-\varphi}{2}\cos\tfrac{\theta+\varphi}{2},$$

(7) $$\cos\theta + \cos\varphi \quad = 2\cos\tfrac{\theta+\varphi}{2}\cos\tfrac{\theta-\varphi}{2},$$

(8) $$\cos\theta - \cos\varphi \quad = -2\sin\tfrac{\theta+\varphi}{2}\sin\tfrac{\theta-\varphi}{2}.$$

For example to derive (5), we first add (2) and (1) to get

(9) $$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta.$$

Then we let $\alpha = \frac{\theta+\varphi}{2}$ and $\beta = \frac{\theta-\varphi}{2}$ in equation (9), then $\alpha+\beta=\theta$, $\alpha-\beta=\varphi$, and we have

$$\sin\theta + \sin\varphi = 2\sin\frac{\theta+\varphi}{2}\cos\frac{\theta-\varphi}{2}.$$