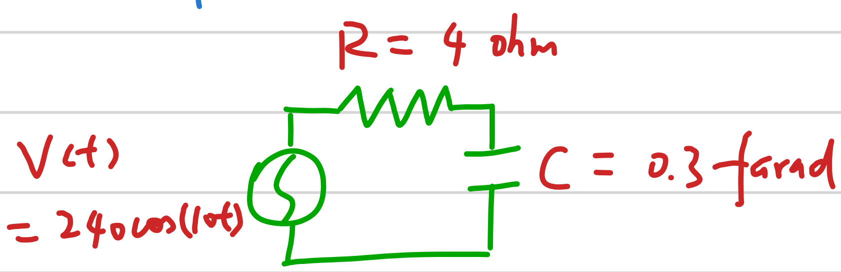
Lecture 10 (Jan. 30)

Problem: A circuit consist of a 4 ohm resistor, a 0.3 farad capacitor, and an AC voltage source supply $V(t) = 240\cos(10t)$ volts

Write down the differential equation for the charge on the capacitor.



$$V_R + V_C = V(t) = 240\cos(10t)$$

$$V_R = RI \quad , \quad I = \frac{dq}{dt} \qquad\qquad R = 4$$

$$V_C = \frac{q}{C} \qquad\qquad C = 0.3$$

$$4q' + \frac{q}{0.3} = 240\cos(10t)$$

$$q' + \frac{q}{1.2} = 60\cos(10t)$$

Review. Direction field.

given a direction, tell which equation is associated?

tell some behaviors of solutions

Sketch direction field for autonomous equations

$$\frac{dy}{dt} = f(y)$$

+ equilibrium solutions

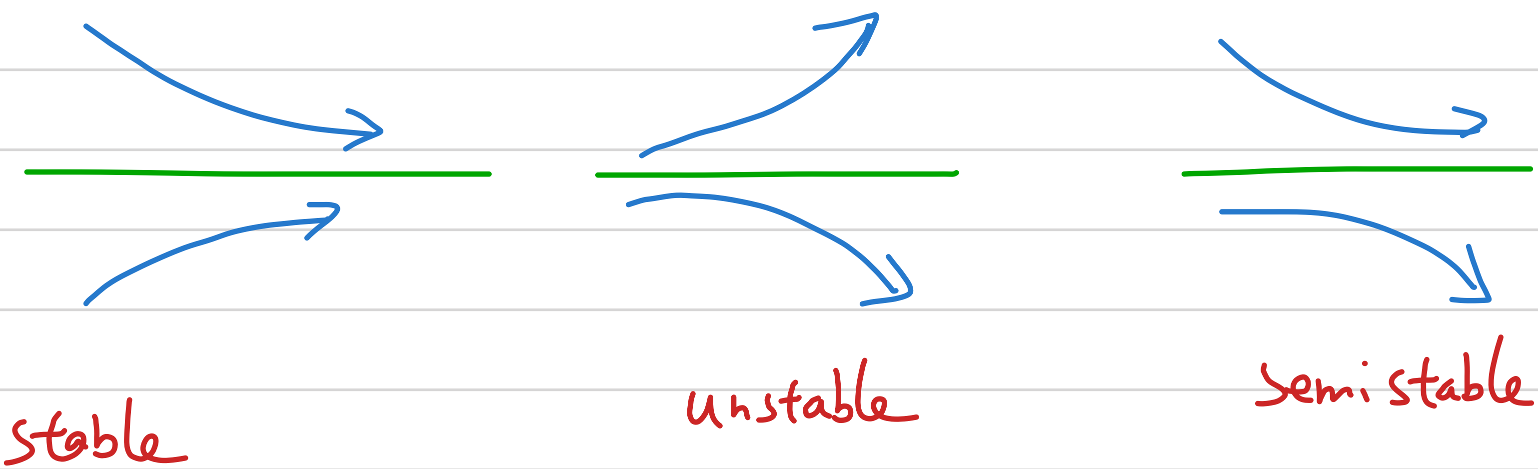
For $\frac{dy}{dt} = f(t,y)$, $y \equiv c$ is an equilibrium solution if

$f(t,c) = 0$ for all t.

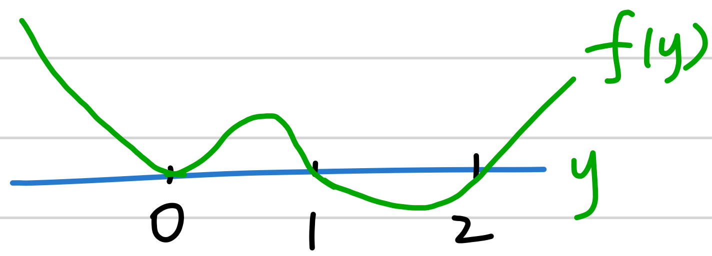
For $\frac{dy}{dt} = f(y)$. . . . . if

$f(c) = 0$

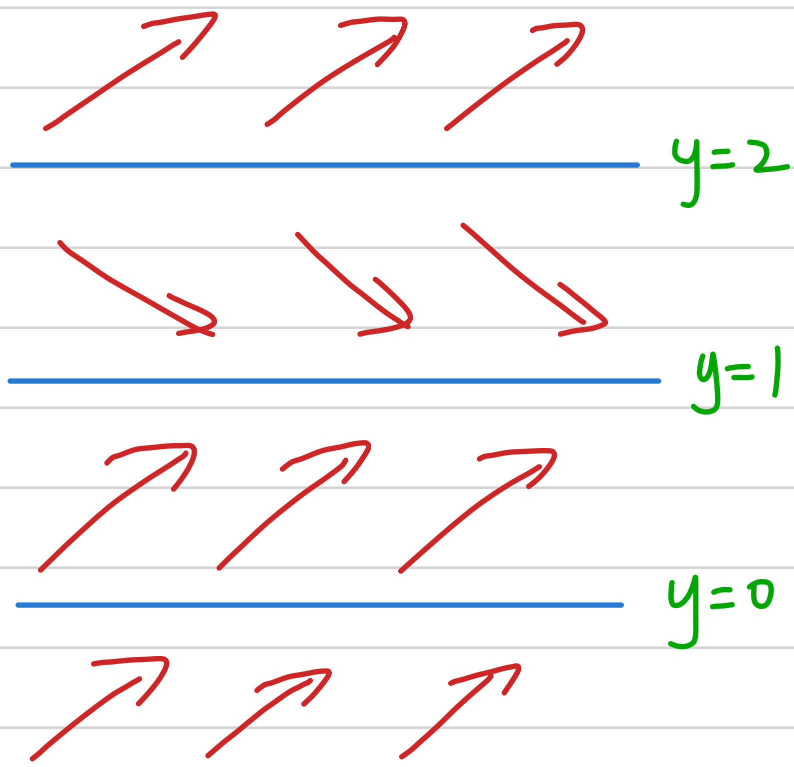
Stable and unstable equilibrium



Example $y' = y^2(y-1)(y-2)$



$f(y) = y^2(y-1)(y-2)$



unstable

stable

semistable

Solve separable D.E.

$$\frac{dy}{dt} = F(t)G(y)$$

$$\frac{dy}{G(y)} = F(t)\,dt$$

Integrate both sides

$$H_1(y) = H_2(t) + C$$

Use I.C to determine C

Solve y as a function of t

Solve Linear D.E.

$$y' + p(t)y = g(t)$$

Method of integrating factors

$$\mu(t) = e^{\int p(t)dt}$$

$$\mu(t)y' + \mu(t)p(t)y = \mu(t)g(t)$$

$$(\mu(t)y)' = \mu(t)g(t)$$

Integrate $\quad \mu(t)y = \int \mu(t)g(t)\,dt + C$

Modeling with first order differential equations

- $F = ma$
- Mixing Problem

$$\frac{ds}{dt} = \text{rate in} - \text{rate out}$$

$S \sim$ amount of salt in tank

- Population growth (logistic model)

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y \qquad \text{autonomous eq.}$$

two equilibriums $\quad y = K, \quad y = 0$

- Newton's law of cooling

$$\frac{dT}{dt} = k(T_A - T) \qquad k > 0$$

Euler's method.

step size h

$$t_{n+1} = t_n + h$$

$$A_n = f(t_n, y_n)$$

$$y_{n+1} = y_n + hA_n$$