Lecture 26 (March 15)

Review.

1st order D.E.

Linear equations. $y' + p(t)y = g(t)$

solve by the method of integrating factors

separable equations implicit/explicit solution

autonomous equation

$y' = f(y)$

equilibrium solutions stable/unstable/semistable

direction fields

logistic model

Modeling: free falling object.

mixing problem

Euler's method.

2nd order D.E.

constant coefficients

$$ay'' + by' + cy = f(t)$$

homogeneous eqn: $f(t) = 0$

characteristic eqn.

$$ar^2 + br + c = 0$$

$b^2 - 4ac$: > 0

< 0

$= 0$.

inhomogeneous eqn — method of undetermined coefficient.

$t^2 + 1$ $\quad Y(t) = at^2 + bt + c$

$2\cos t$ $\quad Y(t) = A\cos t + B\sin t$

e^{3t} $\quad Y(t) = Ae^{3t}$

$\boxed{\text{multiplication by } t}$ when $Y(t)$ doesn't work

Harmonic oscillator

$$m y'' + \gamma y' + k y = \underbrace{f(t)}_{\text{driving force}}$$

$f = 0$ (unforced)

$\gamma^2 - 4mk > 0$ over-damped

< 0 under-damped

$= 0$ critically damped.

underdamped : $r = -\frac{\gamma}{2m} \pm i \underbrace{\frac{\sqrt{4km - \gamma^2}}{2m}}$

w : quasi-frequency.

$f \neq 0$ (forced)

$\gamma = 0$: $f = F_0 \cos wt$.

(beats) $y = A \sin \frac{w + w_0}{2} t \sin \frac{w - w_0}{2} t$ $\quad w \neq w_0$

(resonance) $y = A t \sin w_0 t$ $\quad w = w_0$

$$\gamma \neq 0. \quad y = \underbrace{C_1 y_1 + C_2 y_2}_{\text{transient}} + \underbrace{Y(t)}_{\text{steady-state soln}}$$

Laplace transform

$$\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t)\,dt$$

Calculate Laplace transform & Inverse Laplace transform

- use table of Laplace Transform
- exponential shift formula
- time delay.
- partial fraction decomposition

Step function (Heaviside functions)

$$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t > c \end{cases}$$

Write piecewisely defined function in terms of Heaviside functions

$$f(t) = \begin{cases} t & t < 1 \\ e^{-t} & 1 < t < 2 \\ 1 & t > 2 \end{cases}$$

delta functions

$\delta(t) \longrightarrow 1$

$\delta(t-c) \longrightarrow e^{-sc}$

Convolution integral.

$$(f*g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t f(\tau)g(t-\tau)d\tau$$

$$\mathcal{L}\{(f*g)(t)\} = F(s)G(s)$$

Write solutions of IVP as convolution integrals

$ay''+by'+cy = g(t), \quad y(0)=y_0, \quad y'(0)=v_0$

- transfer function $H(s) = \dfrac{1}{as^2+bs+c}$

- $y(t) = \underbrace{(h*g)(t)}_{}$ $+ \cdots$

write it as convolution integral