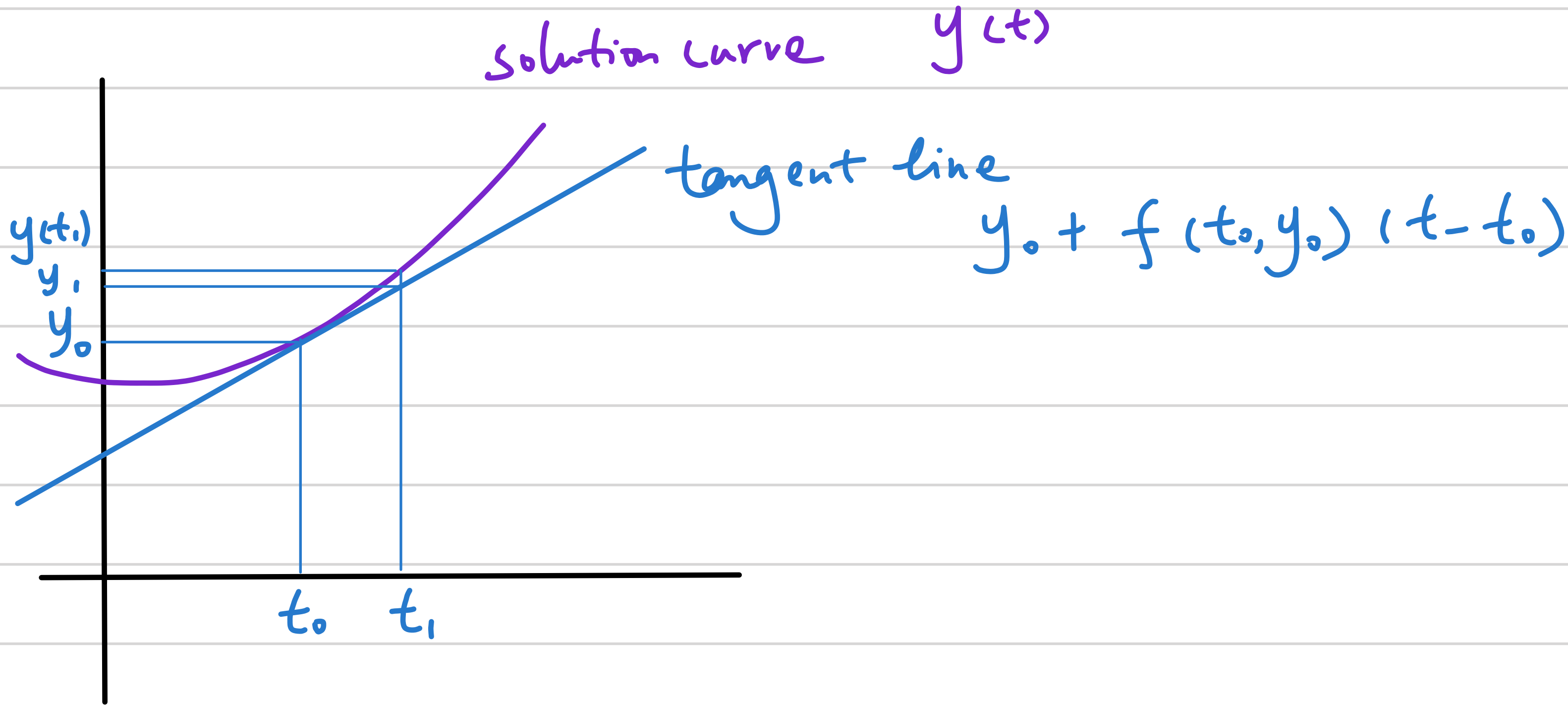
Lecture 9 (Jan. 28)

First order DE:

$$\frac{dy}{dt} = f(t,y)$$
$$y(t_0) = y_0$$



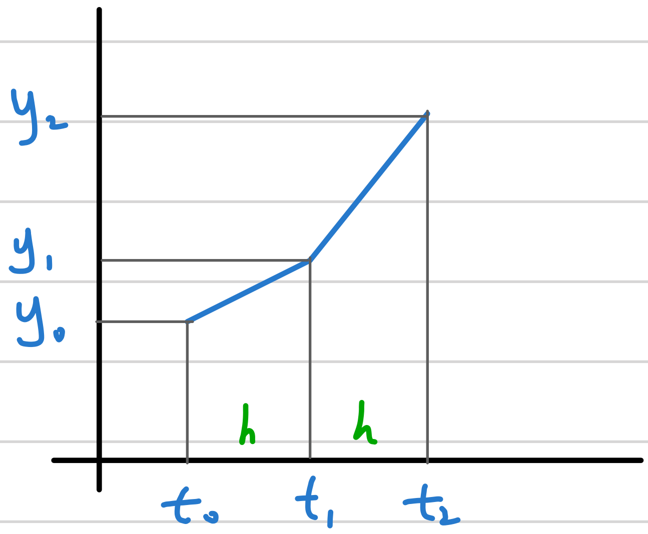
$$y_1 = y_0 + f(t_0, y_0)(t_1 - t_0) \approx y(t_1)$$

if t_1 is close to t_0

Linear approximation of $y(t)$ in a neighborhood of t_0 is good enough!

Euler's method ($f(t,y)$ continuous)

— piecewise linear approximation of the solution $y(t)$.



Step size h

$$t_1 = t_0 + h$$

$$y_1 = y_0 + f(t_0, y_0)(t_1 - t_0)$$

$$= y_0 + hf(t_0, y_0)$$

$$t_2 = t_1 + h$$

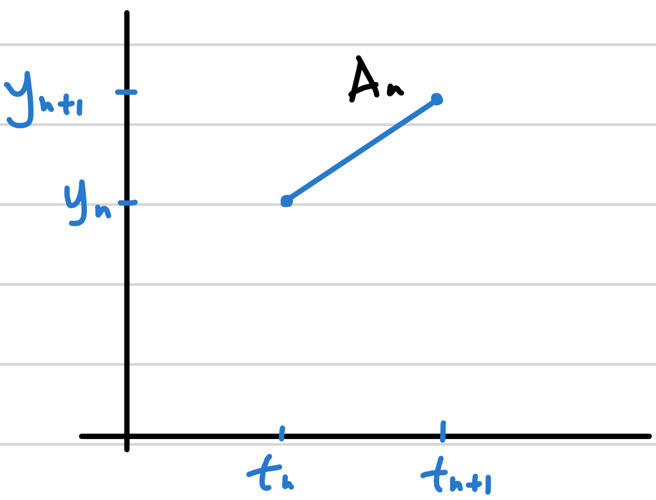
$$y_2 = y_1 + f(t_1, y_1)h$$

$$y_1 \approx y(t_1)$$

$$f(t_1, y_1) \approx f(t_1, y(t_1))$$

$$y_2 \approx y(t_1) + f(t_1, y(t_1))(t_2 - t_1)$$

$$\approx y(t_2)$$



$$t_{n+1} - t_n = h$$

$$A_n = f(t_n, y_n)$$

$$y_{n+1} - y_n = A_n h$$

Euler equations.

$$t_{n+1} = t_n + h$$

$$A_n = f(t_n, y_n)$$

$$y_{n+1} = y_n + hA_n$$

Example: $y' = t(4-y)$, $y(0)=0$, $h=\frac{1}{2}$

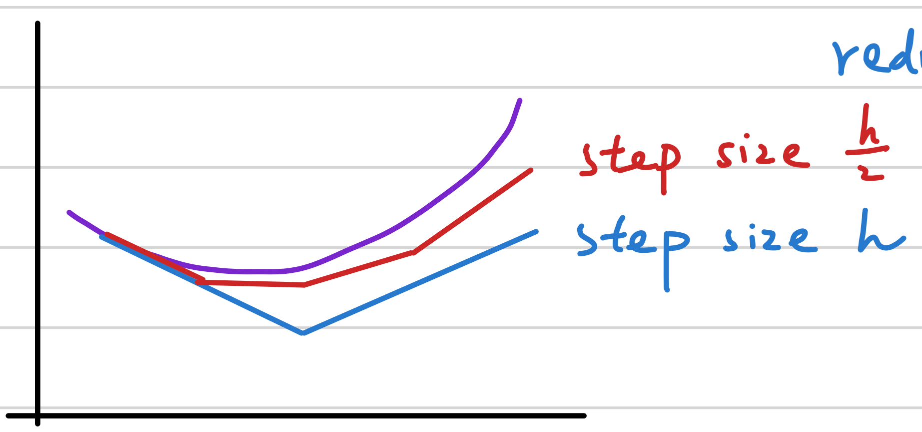
n	t_n	y_n	A_n	hA_n
0	0	0	0	0
1	$\frac{1}{2}$	0	2	1
2	1	1	3	$\frac{3}{2}$
3	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{9}{4}$	$\frac{9}{8}$
4	2	$\frac{29}{8}$	$\times$	$\times$

$y = 4 - 4e^{-t^2/2}$

t_n	y_n	$y(t_n)$
0	0	0
0.5	0	0.47
1	1	1.58
1.5	2.5	2.70
2	3.63	3.46

Error of Euler's method $\sim C_1 h$

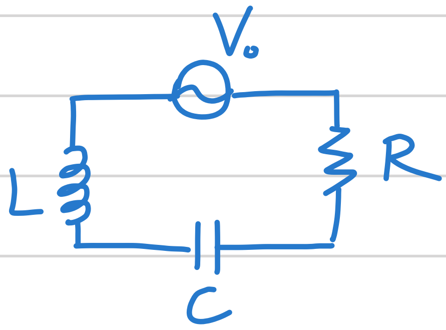
How to lower the error — one easy way: reduce h



Electric Circuits (Long version)

$I(t)$: current

$q(t)$: charge on the capacitor



(~) : electric power, voltage $V_0(t)$

/\/\/\ : resistor, resistance R : constant

Voltage drop $V_R(t) = RI(t)$

=|= : capacitor, with capicitance C : constant

Voltage drop : $V_C(t) = \frac{q(t)}{C}$, $\frac{dq}{dt} = I$

(coil) : inductor : magnetic field due to current

changing current → changing magnetic field

→ generate new voltage

$$V_L(t) = L\frac{dI(t)}{dt}$$

L: inductance, constant   unit: Henries

Kirchhoff's Law: $-V_0 + V_R + V_C + V_L = 0$

q: unit: Coulomb

$$I = \frac{dq}{dt}$$ unit: ampere $= \frac{\text{coulomb}}{\text{second}}$ (current charging/decharging the capacitor)

$$V_L = L\frac{dI}{dt}$$ unit: volt $= \text{henry} \cdot \frac{\text{ampere}}{\text{second}}$

$$V_R = RI$$ unit: volt $= \text{ohm} \cdot \text{ampere}$

$$V_C = \frac{q}{C}$$ unit: volt $= \frac{\text{coulomb}}{\text{farad}}$